

**Exercise 16**

Find the limit.

$$\lim_{x \rightarrow -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4}$$

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**Solution**

Start by making the limit go to infinity instead by making the substitution,  $u = -x$ . Then as  $x \rightarrow -\infty$ ,  $u \rightarrow \infty$ .

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4} &= \lim_{u \rightarrow \infty} \frac{1 - 2(-u)^2 - (-u)^4}{5 + (-u) - 3(-u)^4} \\ &= \lim_{u \rightarrow \infty} \frac{1 - 2u^2 - u^4}{5 - u - 3u^4} \\ &= \lim_{u \rightarrow \infty} \frac{1 - 2u^2 - u^4}{5 - u - 3u^4} \cdot \frac{\frac{1}{u^4}}{\frac{1}{u^4}} \\ &= \lim_{u \rightarrow \infty} \frac{\frac{1}{u^4} - \frac{2}{u^2} - 1}{\frac{5}{u^4} - \frac{1}{u^3} - 3} \\ &= \frac{0 - 0 - 1}{0 - 0 - 3} \\ &= \frac{1}{3} \end{aligned}$$